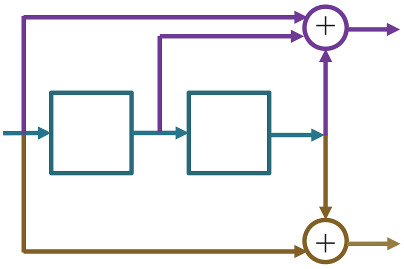
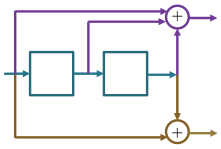
Direct Minimum Distance Decoding

- Suppose $\underline{y} = [11\ 01\ 11]$.
- Find $\hat{\underline{\mathbf{b}}}$.
  - Find the message $\hat{\underline{\mathbf{b}}}$ which corresponds to the (valid) codeword $\hat{\underline{\mathbf{x}}}$ with minimum (Hamming) distance from $\underline{y}$.
    - $\hat{\underline{\mathbf{x}}} = \arg\min_{\underline{\mathbf{x}}} d(\underline{\mathbf{x}}, \underline{\mathbf{y}})$

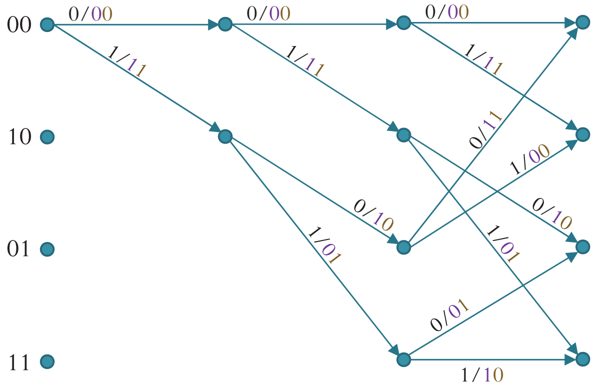


Direct Minimum Distance Decoding



- Suppose $\underline{y} = [\underline{11}\ \underline{01}\ \underline{11}]$.
- Find $\hat{\underline{\mathbf{b}}}$. → 3 bits
  - Find the message $\hat{\underline{\mathbf{b}}}$ which corresponds to the (valid) codeword $\hat{\underline{\mathbf{x}}}$ with minimum (Hamming) distance from $\underline{y}$.

$\underline{y} = [\quad 11 \qquad 01 \qquad 11 \qquad].$



For 3-bit message,
there are $2^3 = 8$ possible codewords.
We can list all possible codewords.
However, here, let's first try to work on the distance directly.

Direct Minimum Distance Decoding

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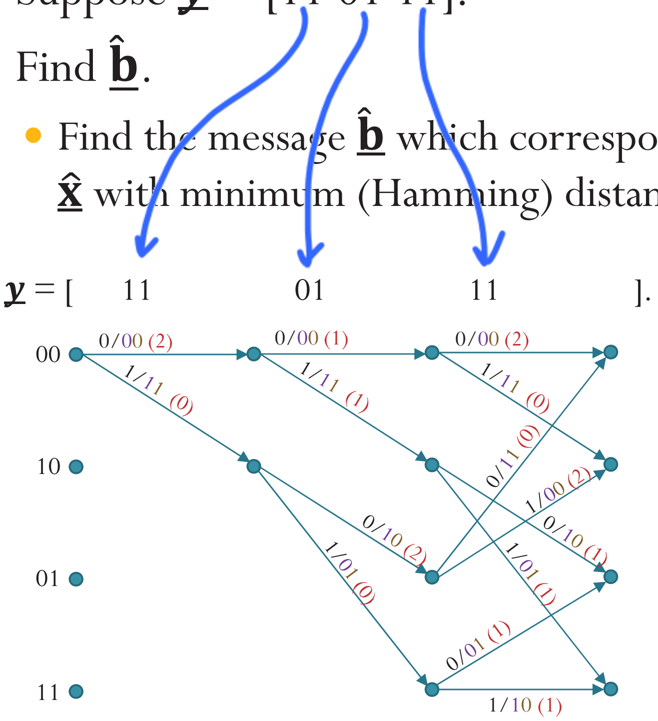

The number in parentheses on each branch is the branch metric, obtained by counting the differences between the encoded bits and the corresponding bits in $\underline{\mathbf{y}}$.

Direct Minimum Distance Decoding

- Suppose $\underline{\mathbf{y}}$ = [11 01 11].
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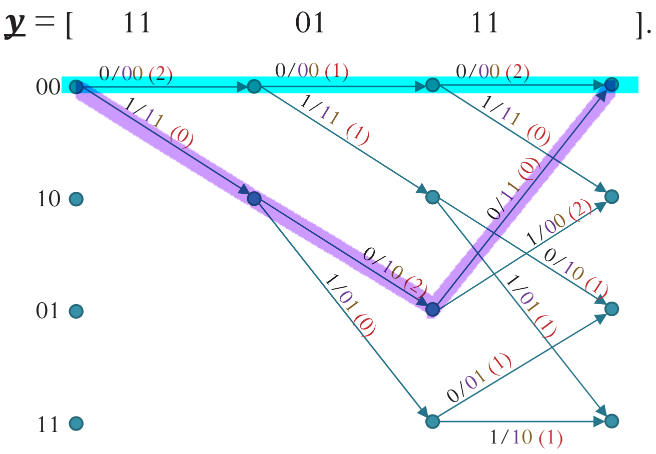


$\underline{\mathbf{b}}$	$d(\underline{\mathbf{x}},\underline{\mathbf{y}})$
000	2+1+2 = 5
001	2+1+0 = 3
010	2+1+1 = 4
011	2+1+1 = 4
100	0+2+0 = 2
101	0+2+2 = 4
110	0+0+1 = 1
111	0+0+1 = 1

min

two answers

Viterbi decoding

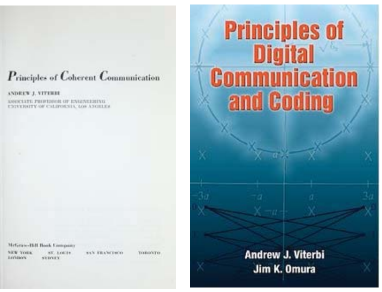
- Developed by Andrew J. Viterbi
  - Also co-founded Qualcomm Inc.
- Published in the paper "Error Bounds for Convolutional Codes and an Asymptotically Optimum Decoding Algorithm", IEEE Transactions on Information Theory, Volume IT-13, pages 260-269, in April, **1967**.

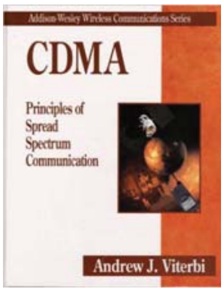
https://en.wikipedia.org/wiki/Andrew_Viterbi

Viterbi and His Decoding Algorithm

[http://viterbi.usc.edu/about/viterbi/viterbi_video.htm]

Andrew J. Viterbi





- 1991: Claude E. Shannon Award
- 1952-1957: MIT BS & MS
  - Studied electronics and communications theory under such renowned scholars as Norbert Wiener, Claude Shannon, Bruno Rossi and Roberto Fano.
- 1962: Earned one of the first doctorates in electrical engineering granted at the University of Southern California (USC)
  - Ph.D. dissertation: error correcting codes
- 2004: USC Viterbi School of Engineering

  named in recognition of his $52 million gift

USC University of Southern California

USC Viterbi School of Engineering

Andrew J. Viterbi

[http://viterbi.usc.edu/about/viterbi/viterbi_video.htm]

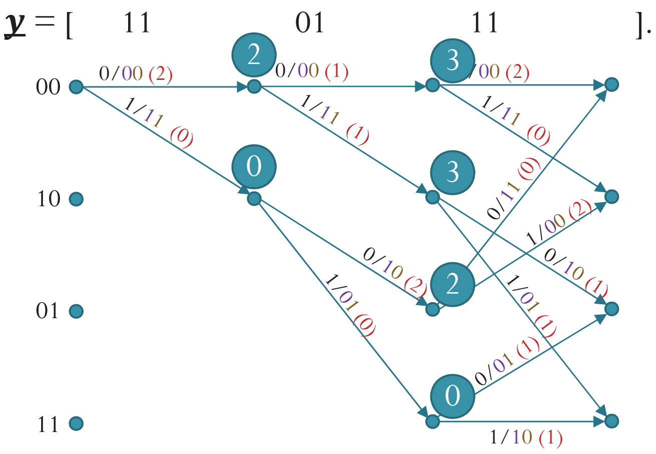
Andrew J. Viterbi

- Cofounded Qualcomm
- Helped to develop the CDMA standard for cellular networks.
- 1998 Golden Jubilee Award for Technological Innovation
  - To commemorate the 50th Anniversary of Information Theory
  - Given to the authors of discoveries, advances and inventions that have had a profound impact in the technology of information transmission, processing and compression.

1. Norman Abramson: For the invention of the first random-access communication protocol.
2. Elwyn Berlekamp: For the invention of a computationally efficient algebraic decoding algorithm.
3. Claude Berrou, Alain Glavieux and Punya Thitimajshima: For the invention of turbo codes.
4. Ingrid Daubechies: For the invention of wavelet-based methods for signal processing.
5. Whitfield Diffie and Martin Hellman: For the invention of public-key cryptography.
6. Peter **Elias**: For the invention of convolutional codes.
7. G. David Forney, Jr: For the invention of concatenated codes and a generalized minimum-distance decoding algorithm.
8. Robert M. Gray: For the invention and development of training mode vector quantization.
9. David **Huffman**: For the invention of the Huffman minimum-length lossless data-compression code.
10. Kees A. Schouhamer Immink: For the invention of constrained codes for commercial recording systems.
11. Abraham Lempel and Jacob Ziv: For the invention of the Lempel-Ziv universal data compression algorithm.
12. Robert W. Lucky: For the invention of pioneering adaptive equalization methods.
13. Dwight O. North: For the invention of the matched filter.
14. Irving S. Reed: For the co-invention of the Reed-Solomon error correction codes.
15. Jorma Rissanen: For the invention of arithmetic coding.
16. Gottfried Ungerboeck: For the invention of trellis coded modulation.
17. Andrew J. **Viterbi**: For the invention of the Viterbi algorithm.

Viterbi Decoding: Ex. 1

- Suppose $\underline{\boldsymbol{y}}$ = [11 01 11].
- Find $\hat{\underline{\mathbf{b}}}$.
  - Find the message $\hat{\underline{\mathbf{b}}}$ which corresponds to the (valid) codeword $\hat{\underline{\mathbf{x}}}$ with minimum (Hamming) distance from $\underline{\boldsymbol{y}}$.



Each **circled number** at a node is the running (cumulative) path metric, obtained by summing branch metrics (distance) up to that node. Here, it is simply the cumulative distance.

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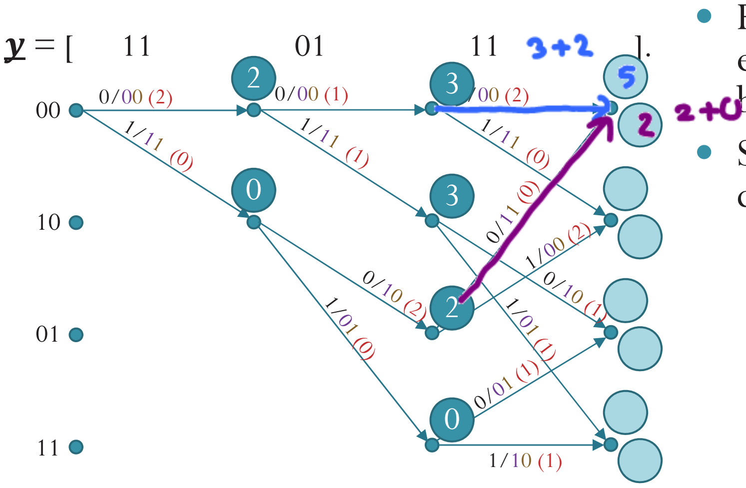

- For the last column of nodes, each of the nodes has two branches going into it.
- So, there are two possible cumulative distance values.

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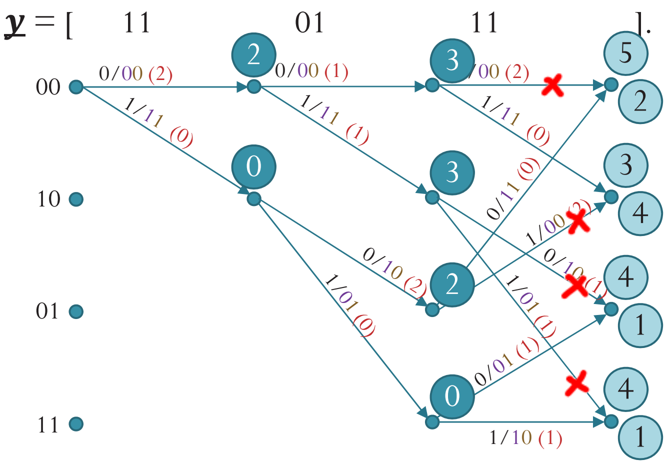

min(~~3+2~~, 2+0)

min(3+0, ~~2+2~~)

min(~~3+1~~, 0+1)

min(~~3+1~~, 0+1)

We **discard the larger-distance path** because, regardless of what happens subsequently, this path will have a larger Hamming distance from $\underline{\mathbf{y}}$.

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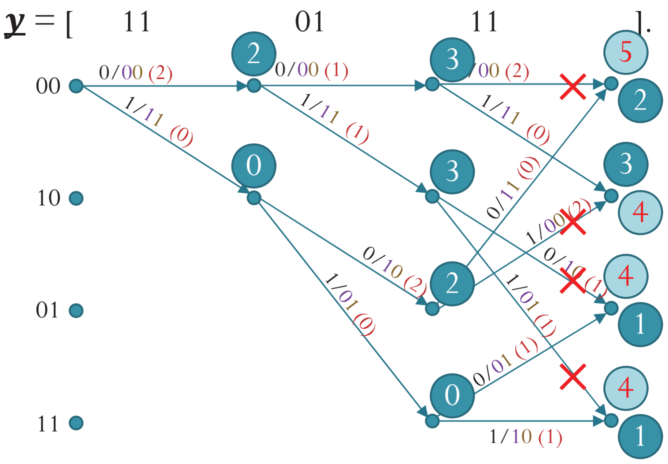

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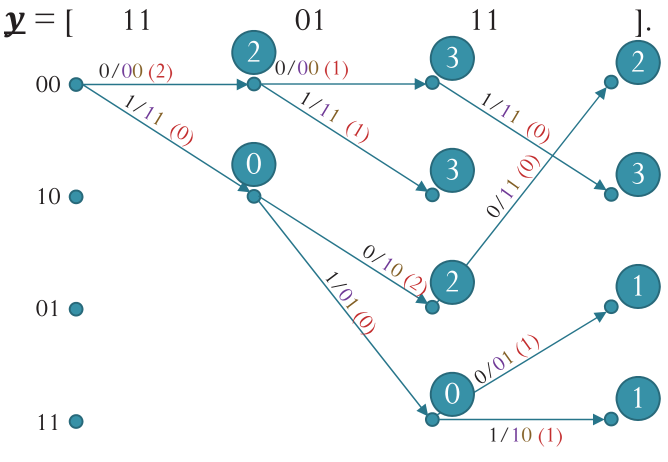

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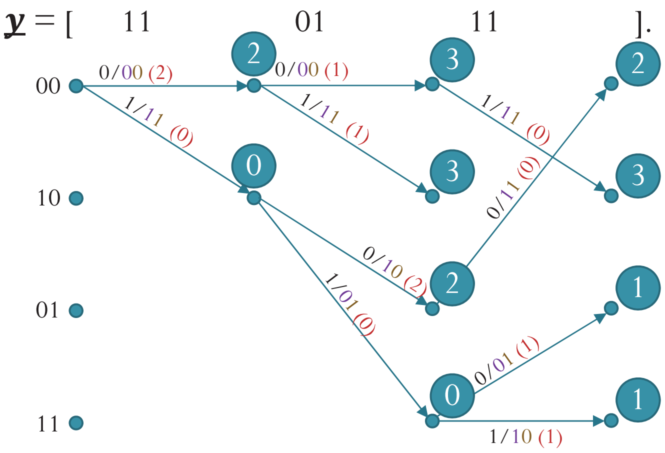

Note that we keep exactly one (optimal) **survivor path** to each state. (Unless there is a tie, then we keep both or choose any.)

Viterbi Decoding: Ex. 1

- Suppose $\underline{y}$ = [11 01 11].
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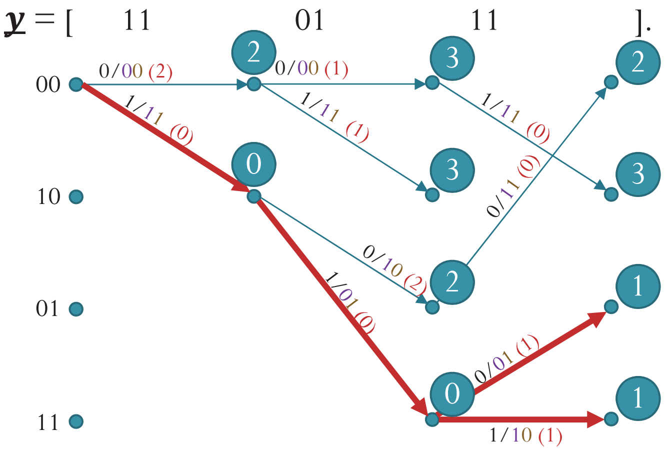
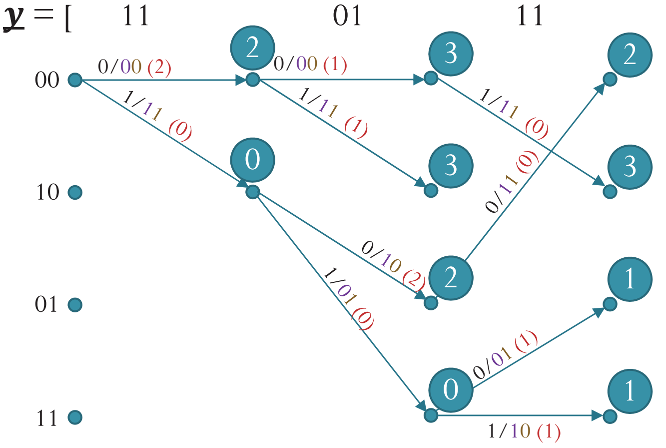


- So, the codewords which are nearest to $\underline{y}$ is [11 01 01] or [11 01 10]. } $\hat{\underline{x}}$
- The corresponding messages are [110] or [111], respectively. } $\hat{\underline{b}}$

Viterbi Decoding: Ex. 2

- Suppose $\underline{\boldsymbol{y}} = [\underbrace{11\ 01\ 11}_{\text{same as before}}\ 00\ 01\ 10]$.
- Find $\hat{\underline{\mathbf{b}}}$.

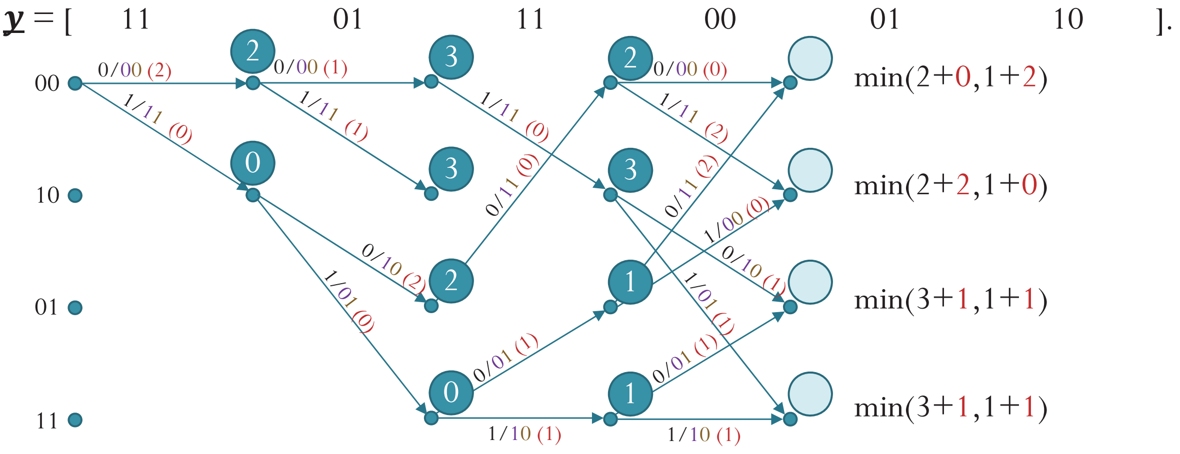
$\underline{\boldsymbol{y}} = [\quad 11 \quad 01 \quad 11 \quad 00 \quad 01 \quad 10 \quad].$



The first part is the same as before. So, we simply copy the diagram that we had.

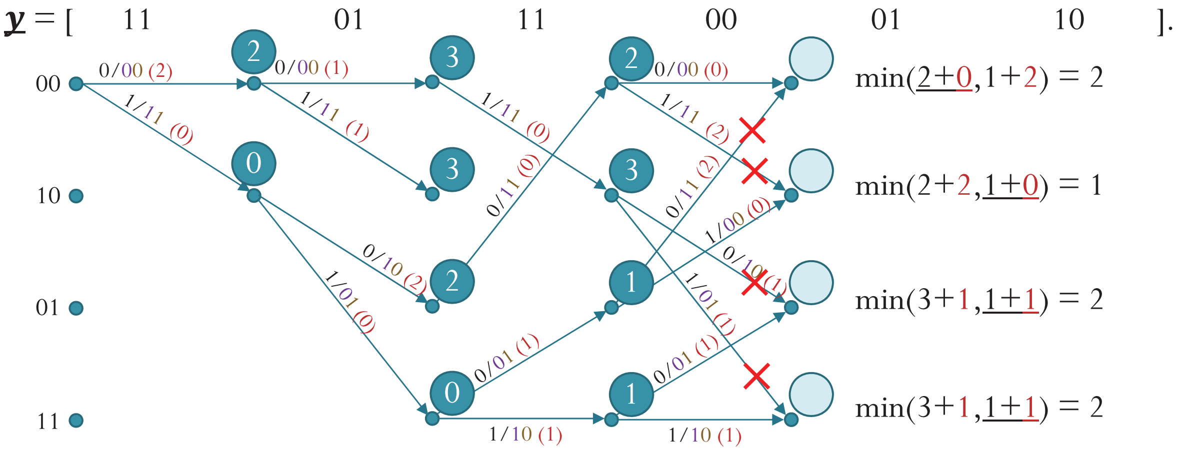
Viterbi Decoding: Ex. 2

- Suppose $\underline{\boldsymbol{y}} = [\underbrace{11\ 01\ 11}_{\text{same as before}}\ 00\ 01\ 10]$.
- Find $\hat{\underline{\mathbf{b}}}$.



Viterbi Decoding: Ex. 2

- Suppose $\underline{y} = [11\ 01\ 11\ 00\ 01\ 10]$.
- Find $\hat{\underline{\mathbf{b}}}$.



min(2+0,1+2) = 2

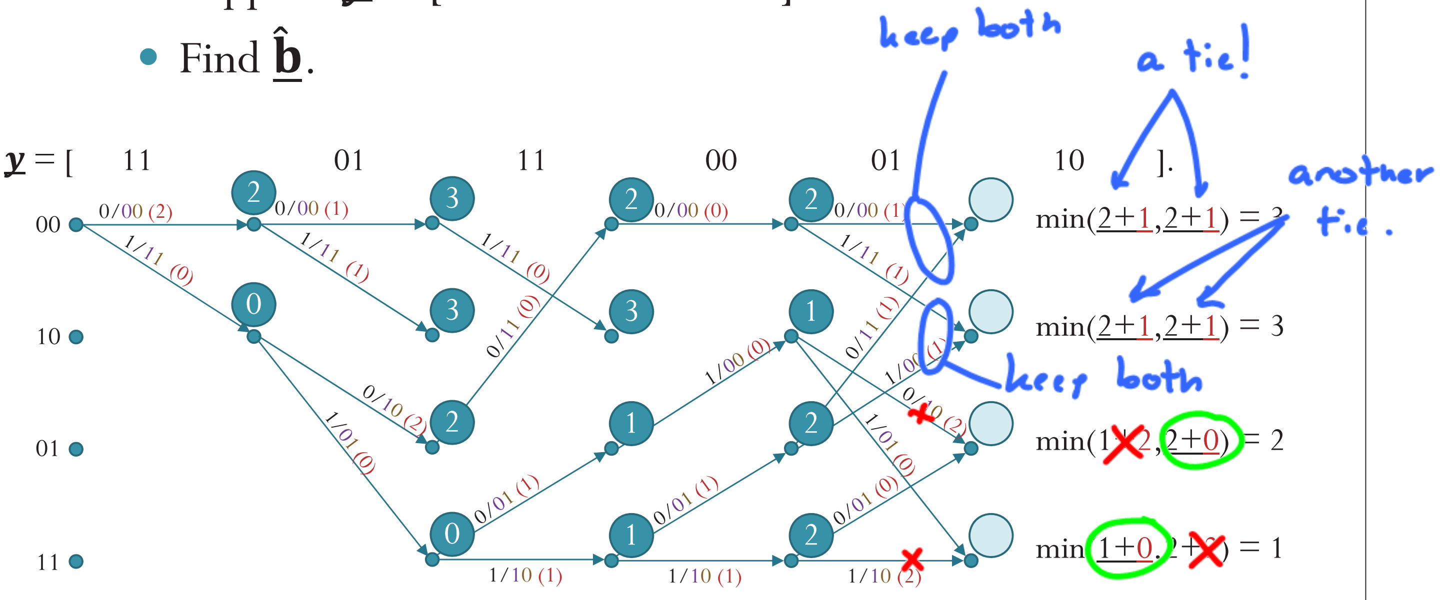
min(2+2,1+0) = 1

min(3+1,1+1) = 2

min(3+1,1+1) = 2

Viterbi Decoding: Ex. 2

- Suppose $\underline{y} = [11\ 01\ 11\ 00\ 01\ 10]$.
- Find $\hat{\underline{\mathbf{b}}}$.



min(2+1,2+1) = 3

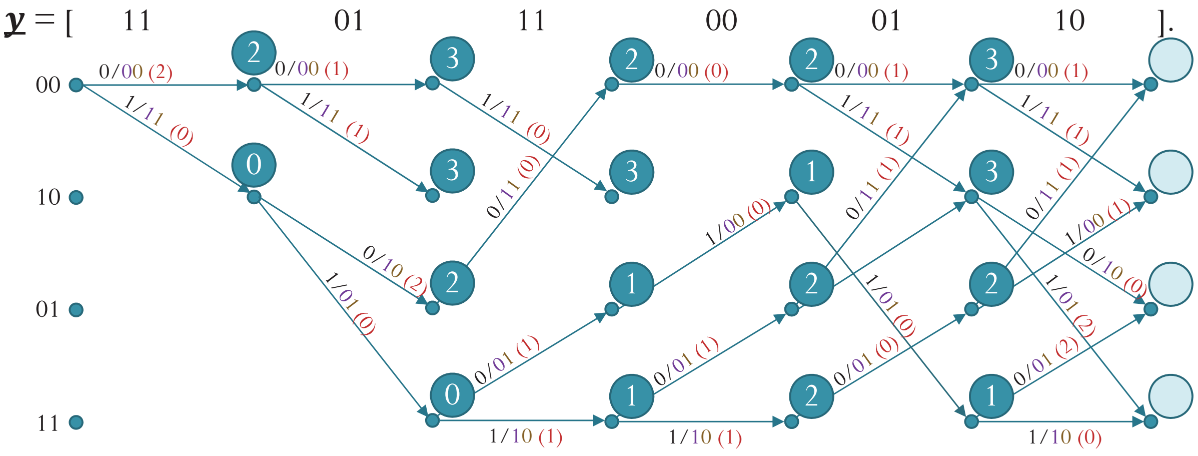
min(2+1,2+1) = 3

min(1+2,2+0) = 2

min(1+0,2+2) = 1

Viterbi Decoding: Ex. 2

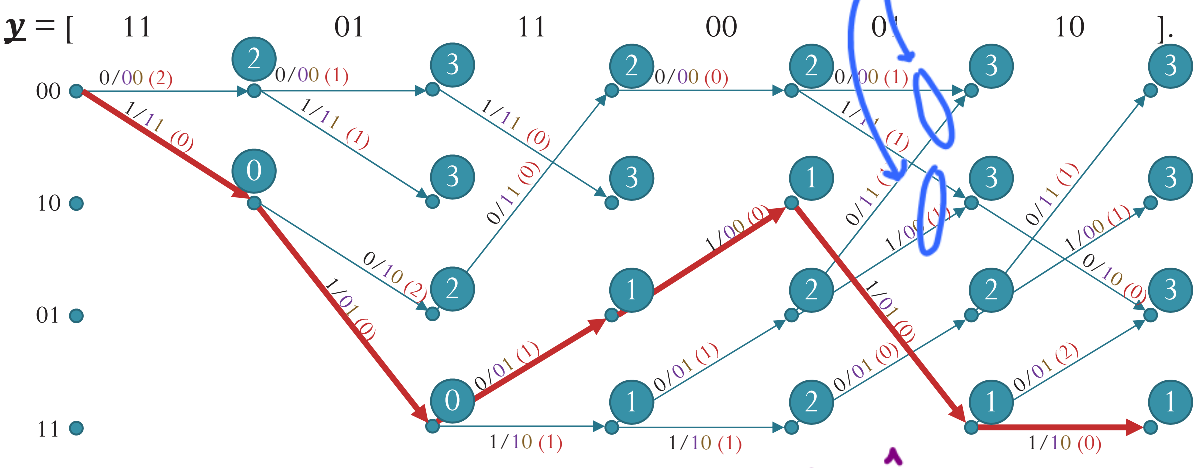
- Suppose $\underline{\boldsymbol{y}} = [11\ 01\ 11\ 00\ 01\ 10]$.
- Find $\hat{\underline{\mathbf{b}}}$.



Viterbi Decoding: Ex. 2

- Suppose $\underline{\boldsymbol{y}} = [11\ 01\ 11\ 00\ 01\ 10]$.
- Find $\hat{\underline{\mathbf{b}}}$.

Turn out the final min. cum. dist, does not contain these. So, we have a unique solution.



$\hat{\underline{\mathbf{x}}} = [11\ 01\ 01\ 00\ 01\ 10]$

$\hat{\underline{\mathbf{b}}} = [1\ 1\ 0\ 1\ 1\ 1]$

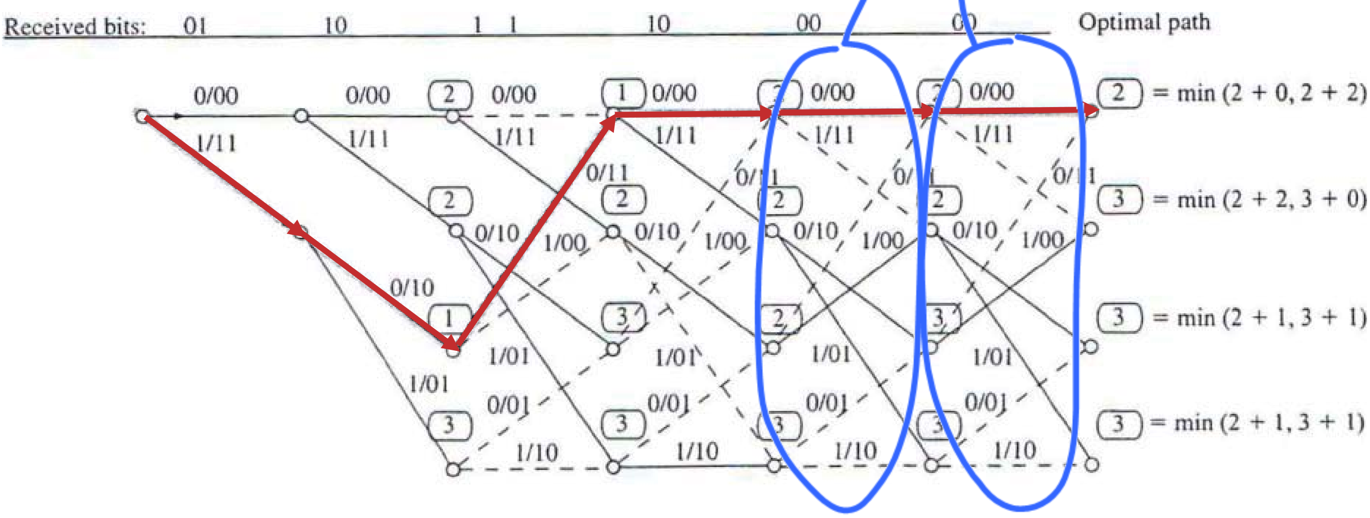
To get $\hat{x}$,
① we first find the path by tracing from the RHS back to the starting state.

② We read $\hat{x}$, $\hat{b}$ from the LHS (starting state).

Viterbi Decoding: Ex. 3

Note that because the bits in the received vector are the same, the distance values in the trellis diagram will also be the same.

- Suppose $\underline{\boldsymbol{y}}$ = [01 10 11 10 00 00].



$\hat{\underline{\mathbf{x}}}$ = [11 10 11 00 00 00]

$\hat{\underline{\mathbf{b}}}$ = [1 0 0 0 0 0]

Reference

- Chapter 15 in [Lathi & Ding, 2009]
- Chapter 13 in [Carlson & Crilly, 2009]
- Section 7.11 in [Cover and Thomas, 2006]